Introduction to quantum information

Separate a mathematical state, a physical system and the information a measurement can reveal.

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By the end, you'll be able to

- Separate the quantum state from an observed outcome.
- Explain why copying classical information differs from copying an unknown quantum state.

Information has a physical representation

A bit may be stored as a voltage. A qubit may use two controlled energy levels of a physical system. The abstract model focuses on distinguishable states and allowed operations rather than one particular device technology.

An ideal pure qubit uses a unit vector in a two-dimensional complex vector space. That mathematical description helps us calculate experimental predictions. It is not a list of values we can freely read from the physical system.

A worked comparison

Consider two states:

$$|+\rangle = \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \qquad |-\rangle = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

Both give 0 and 1 equally often in the Z basis. Yet applying H before measurement makes $|+\rangle$ produce 0 and $|-\rangle$ produce 1. Their relative sign carries a distinction that the first measurement setting does not reveal.

The lesson is practical: a distribution from one basis is not a complete description of an unknown quantum state.

What can be copied?

Known classical basis labels can be copied into another register using reversible logic. An arbitrary unknown quantum state cannot be perfectly copied by one universal operation. This restriction is the no-cloning principle.

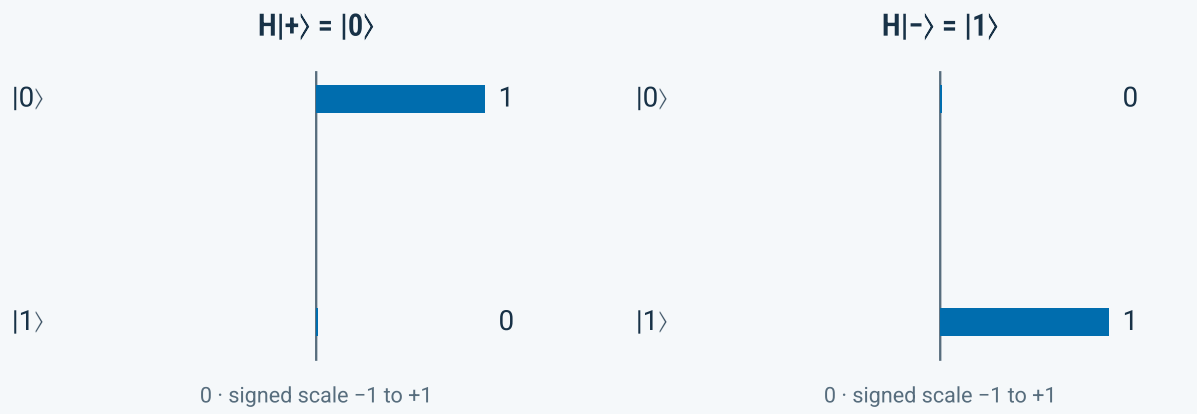
Practise

In the qubit explorer, prepare an equatorial state. Keep its polar angle fixed and vary only relative phase. Record what changes in the sphere and what stays fixed in the Z-basis bars. Then use the interference lab to make the phase difference observable.

What a state description lets you predict

WORKED DIAGRAM · F02

Identical Z counts can hide different states



These are the amplitudes AFTER applying H. Before H, plus and minus both give equal Z counts; afterwards, their outputs are distinct basis states.

The diagram compares two preparations that look identical to a Z-basis detector. The amplitudes have the same magnitudes, but one preparation has a minus sign on its second component. A final H converts that difference into different definite outcomes. Information about a preparation depends on the experiment you choose to perform, not just the number of symbols in its mathematical description.

There are three objects to keep separate. A physical system is the device being controlled. A state is the mathematical object used to predict its behaviour under a specified model. A measurement record is the classical data returned by a particular experiment. In a simulation we can inspect the state because the simulator stores it explicitly. A real qubit does not provide an amplitude-inspection command. Learning a state from observations generally requires many identically prepared systems and more than one measurement setting.

Worked example: distinguish plus from minus

Suppose a trusted source prepares either the plus state or the minus state, and you know those are the only alternatives. A direct Z measurement does not identify the preparation: either case gives zero or one with equal probability. Repeating only that setting improves your estimate of the half-and-half distribution but does not expose the sign. More shots cannot repair a measurement choice that discards the distinction you need.

Instead, apply H before measuring. On the plus state, the two positive components add into the zero output while the one output cancels. On the minus state, the zero output cancels while the one output adds. The final outcomes are therefore zero and one respectively. This perfect discrimination is possible because the two candidate states are orthogonal. It is a property of this carefully stated pair, not a universal method for identifying any unknown qubit from one copy.

Now change the candidate pair to the zero state and the plus state. These are not orthogonal. A Z measurement that returns one rules out the zero preparation, but a zero result can come from either candidate. H followed by Z measurement simply moves the ambiguity elsewhere. The overlap between the states prevents a measurement that always labels both correctly from one copy. You can accept errors or inconclusive results, or gather more preparations; you cannot remove the physical constraint by adding a better user interface.

An information task needs a contract

Before claiming that a quantum procedure transmits, stores or reveals information, specify the alternatives and the permitted resources. Does the sender choose from two orthogonal states or a larger set? Does the receiver know the preparation basis? Can the receiver request repeated copies? Is there classical side information? These details change the task. A claim about one protocol cannot be transferred to another merely because both use one qubit.

For example, a sender can encode a classical bit in the zero and one states and let the receiver measure in Z. The receiver can recover that label perfectly in the ideal model. If the sender instead chooses an arbitrary point on the Bloch sphere, the receiver cannot read its two angles exactly from one measurement. A continuous state description and a limited classical readout are compatible because the description specifies predictions across possible experiments, rather than a list of freely extractable numbers.

Test the distinction using interference

The lab below implements a zero input, H, a phase operation and a final H. At phase zero, the state before the final H is plus. At phase 180 degrees it is minus. First select zero degrees, record the expected final bit, then reveal. Change to 180 degrees, make a new prediction and reveal again. Compare the complex amplitudes with the deterministic output. At 90 degrees, predict whether the same perfect distinction survives or whether both final outcomes become possible.

L03 · INTERACTIVE LAB

Runs on your device

Superposition & interference

The circuit is H → P(φ) → H on |0⟩. Relative phase changes interference at the final H. The intermediate and final Z probabilities need not agree.

MODEL · Ideal pure-state vector, complex double precision ORDER · q0 is the leftmost bit

Initial setup · reference state

Exact probability



Statevector before readout · ideal simulation

Basis	Amplitude a + bi	Magnitude	Phase (rad)	Probability
|0⟩	`1 + 0i`	1.000000	0.000000	100.0%
|1⟩	`0 + 0i`	0.000000	undefined	0.0%

Signs and complex phases are part of the state. Phase is undefined for a zero amplitude. A shared global phase changes no probabilities; relative phases can affect a later operation. Values are rounded to six decimal places.

Prediction to try: At the selected φ, predict the final state or probabilities. Explain which amplitudes will reinforce or cancel.

Challenge: Compare φ = 0°, 90° and 180°. Connect the complex amplitude table to the final distribution.

Open the live experiment studio

The amplitude table is a view into the ideal calculation. It helps explain why the output changes, but it should not be included in the information available to a real receiver unless the protocol explicitly supplies that classical description. Treating simulator internals as experimentally observed data would quietly turn the task into a different, easier problem.

Practice: improve a measurement plan

A student is given many preparations, all either plus or all minus. They collect ten thousand Z measurements and report an almost even split. They conclude the source must be plus because plus “means equal probabilities.” Identify the missing step, and design a setting that answers the actual question.

> Check your reasoning
>
> Minus also has equal Z probabilities. The evidence is consistent with both candidates. Apply H before the Z readout on each fresh preparation. An ideal all-zero result identifies plus; an ideal all-one result identifies minus. A practical implementation would also model preparation and readout errors and report confidence rather than claiming perfect hardware performance. The key change is the measurement basis, not merely the sample size.

This example introduces a habit used throughout quantum information: ask which distinctions the chosen operations make observable. It prepares you for no-cloning, quantum communication and security protocols, where access to a state and knowledge of its preparation are different resources.

Reference: IBM Quantum — Quantum information.

Check your understanding

PAUSE & THINK

Which test perfectly distinguishes plus and minus in the ideal model?

- [ ] More Z measurements without changing basis
- [ ] H followed by Z readout
- [ ] Inspecting one physical amplitude directly

Answer: H followed by Z readout. H maps plus to zero and minus to one. Their direct Z probabilities are identical.