# Understanding qubits

Meet the smallest quantum information system, with a sphere you can explore.

16 min read Dr. Charith Jayasekara

**By the end, you'll be able to**

- Describe a pure qubit using amplitudes and angles.
- Predict computational-basis measurement probabilities.
- Distinguish a mathematical state from one measured bit.

## Two amplitudes, one state

A pure qubit lives in a two-dimensional complex vector space. After removing an unobservable global phase, we can write it as

$$|\psi\rangle = \cos(\theta/2)|0\rangle + e^{i\phi}\sin(\theta/2)|1\rangle.$$

The polar angle $\theta$ sets the Z-basis probabilities. The azimuth $\phi$ sets the relative phase. The explorer uses **degrees** for its controls; the formula uses radians.

## Follow the sphere

At $\theta = 0$, the state is $|0\rangle$. At $\theta = 180^\circ$, it is $|1\rangle$, up to a global phase. Along the equator, $\theta = 90^\circ$ and both Z outcomes have probability $1/2$.

## Work one example

Choose $\theta = 120^\circ$ and $\phi = 90^\circ$. Then

$$|\psi\rangle = \frac{1}{2}|0\rangle + i\frac{\sqrt{3}}{2}|1\rangle.$$

Therefore $P(0) = 1/4$ and $P(1) = 3/4$. A 1,024-shot experiment has expected counts 256 and 768, but its observed counts fluctuate.

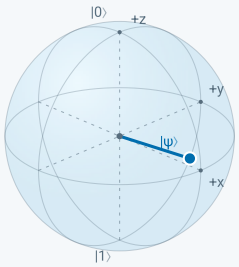
## What the picture does and does not mean

The arrow is a geometric representation of an ideal pure state. It is not a tiny physical arrow inside every device. Mixed states belong inside the Bloch sphere; this particular explorer stays on the surface and does not model decoherence.

## Build the angle representation step by step

WORKED DIAGRAM · F07

### One state, two complementary descriptions




**Polar angle θ:** 120°

**Relative phase φ:** 90°

α = 1/2; β = i√3/2

x = 0, y = √3/2, z = −1/2

At θ = 120° and φ = 90°, α = 1/2 and β = i√3/2. The Bloch vector is (0, √3/2, −1/2); the Z probabilities are 1/4 and 3/4. An orthographic projection can place a surface point inside the circular outline.

A normalised pair of complex amplitudes initially contains four real numbers. Normalisation removes one independent choice, and a common global phase removes another physical distinction. Two real parameters remain for a pure qubit. The polar and azimuthal angles are one way to describe them. They label a state, rather than a tiny physical arrow rotating inside a device.

The half-angle in the amplitude formula is essential. When the polar angle reaches 180 degrees, its half reaches 90 degrees, so the zero amplitude vanishes and the one amplitude has magnitude one. Using the full angle by mistake would put the south-pole label on a state that is equivalent to zero. Check the north pole, equator and south pole whenever translating angles into amplitudes.

## Worked example: away from the equator

Choose a polar angle of 120 degrees and a relative phase of 90 degrees. The half-angle is 60 degrees. Its cosine is one half, and its sine is square root of three over two. The phase factor at 90 degrees is i. The state is therefore

$$|\psi\rangle = \frac{1}{2}|0\rangle + \frac{i\sqrt{3}}{2}|1\rangle.$$

The Z probabilities are one quarter and three quarters. Its Bloch coordinates are zero, square root of three over two, and negative one half. A negative z coordinate means the one result is more likely. The positive y coordinate records the chosen relative phase. The diagram shows this as a point below the equator toward positive y, using the same projection as the live explorer.

Now keep the polar angle fixed and replace the relative phase with 270 degrees. The one amplitude becomes negative imaginary. The Z probabilities remain one quarter and three quarters, while the y coordinate changes sign. A detector measuring only Z cannot distinguish these preparations. A suitable basis change before readout can reveal their difference, because their amplitudes interfere differently.

## The six useful reference states

Learn the two poles and four equatorial directions as landmarks. The poles are zero and one. Positive x is plus, and negative x is minus. Positive y has a positive imaginary one amplitude, while negative y has a negative imaginary one

amplitude. Each equatorial state has equal Z probabilities, so the Z histogram cannot tell which equatorial direction was prepared.

At a pole, the relative phase angle becomes redundant. If the one amplitude is zero, changing its assigned phase changes nothing. At the south pole, a phase on the sole nonzero component is only global. This is a coordinate limitation, like longitude at a geographic pole, rather than an inconsistency in the state model. Numerical phase displays should not attach physical meaning to a zero component.

## What a gate changes

An X gate exchanges the amplitudes. On the sphere it performs a half turn about x. Starting at the north pole, it reaches the south pole. Starting at plus, it leaves the physical state unchanged because plus is an eigenstate of X. A gate's name alone therefore does not imply the same visible change for every input.

Z adds a relative sign to the one component. It leaves both Z probabilities unchanged while exchanging plus and minus. H exchanges the roles of the x and z directions and reverses y. This gives a useful visual check: H sends north to positive x, positive x to north, and positive y to negative y. Use the signed coordinates and amplitudes to verify these transformations instead of trusting a perspective drawing by itself.

## A controlled exploration

The lab starts at zero. First predict its amplitudes and reveal the result. Then set the polar angle to 120 degrees and relative phase to 90 degrees, enter the prediction calculated above and reveal again. Check the complex entries, probabilities and coordinates together. Changing a control hides the result until you record a new prediction.

L01 · INTERACTIVE LAB

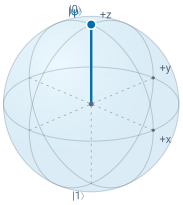
Runs on your device

### Qubit explorer

Prepare a pure qubit, apply gates, then compare amplitudes, phases and Z-basis probabilities. Reset restores the initial preparation shown by the lesson's controls.

MODEL · Ideal pure-state vector, complex double precision  ORDER · q0 is the leftmost bit

### Initial setup · reference state




Exact probability

|0⟩ 100.0%

|1⟩ 0.0%

**Statevector before readout · ideal simulation**

| Basis | Amplitude a + bi | Magnitude | Phase (rad) | Probability |
|---|---|---|---|---|
| \|0⟩ | `1 + 0i` | 1.000000 | 0.000000 | 100.0% |
| \|1⟩ | `0 + 0i` | 0.000000 | undefined | 0.0% |

Signs and complex phases are part of the state. Phase is undefined for a zero amplitude. A shared global phase changes no probabilities; relative phases can affect a later operation. Values are rounded to six decimal places.

Prediction to try: Predict the amplitudes and Z-basis probabilities for the preparation and gates you have chosen. If starting from |0⟩, what will H do?

Challenge: Prepare θ = 120°, φ = 90°. Predict both complex amplitudes. Then change φ without changing θ.

Open the live experiment studio

Next, compare phases zero, 90 and 180 degrees at polar angle 90 degrees. The histogram should remain half-and-half while the arrow and amplitude phases move. Finally, apply H separately to each preparation and compare the new distributions. Reset and re-enter the angles before each trial so that you do not accidentally accumulate gates from a previous experiment.

## Practice: identify an incomplete description

You are told that a pure qubit gives one with probability three quarters in Z. Can you determine its relative phase? Can you determine its polar angle in the range zero to 180 degrees? State what additional experiment might reveal the missing information.

**Check your reasoning**

The polar angle is 120 degrees because the squared sine of its half is three quarters. The relative phase is undetermined by Z probabilities. Measurements in X and Y on fresh preparations can estimate the other Bloch components and hence constrain the phase. The single Z statistic describes a whole circle of pure states, not one unique state.

Reference: Microsoft Learn — The qubit.

## Check your understanding

PAUSE & THINK

### A pure state gives P(1) = 3/4 in Z. What does that determine?

- ☐ Both angles uniquely
- ☐ Polar angle 120°, with relative phase still unknown
- ☐ Relative phase 120°, with polar angle unknown

**Answer:** Polar angle 120°, with relative phase still unknown. The Z probabilities fix θ in the chosen range, but are independent of φ.

PAUSE & THINK

### You change only φ while keeping θ fixed. What stays the same?

- ☐ The entire quantum state
- ☐ The Z-basis measurement probabilities
- ☐ All possible measurement results in every basis

**Answer:** The Z-basis measurement probabilities. $P(0) = \cos^2(\theta/2)$ and $P(1) = \sin^2(\theta/2)$ do not contain φ. Other measurement settings can reveal the relative phase.