The Bloch sphere

Read a geometric picture without mistaking it for the physical device.

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By the end, you'll be able to

- Convert qubit angles into Bloch coordinates.
- Explain the pure-state limitation of the explorer.

Coordinates for a pure qubit

For the angle representation in F07, the Bloch coordinates are

$$x = \sin\theta\cos\phi, \qquad y = \sin\theta\sin\phi, \qquad z = \cos\theta.$$

They satisfy $x^2 + y^2 + z^2 = 1$. In terms of amplitudes, $x = 2\,\mathrm{Re}(\alpha^*\beta)$, $y = 2\,\mathrm{Im}(\alpha^*\beta)$ and $z = |\alpha|^2 - |\beta|^2$. The north pole is $|0\rangle$, south is $|1\rangle$, positive x is $|+\rangle$, and positive y is $(|0\rangle + i|1\rangle)/\sqrt{2}$.

A worked point

At $\theta = 90^\circ$ and $\phi = 90^\circ$, the coordinates are $(0, 1, 0)$. The Z probabilities are $P(0) = (1+z)/2 = 1/2$ and $P(1) = (1-z)/2 = 1/2$.

Move ϕ around the equator. The arrow direction changes while z stays zero. This makes it easy to see why a single Z measurement setting misses relative phase.

What lies inside?

Mixed single-qubit states have Bloch vectors of length at most one. The maximally mixed state sits at the centre. It is different from an equatorial pure state even though both can have equal Z probabilities. Describing mixed states properly requires density matrices, planned in X01.

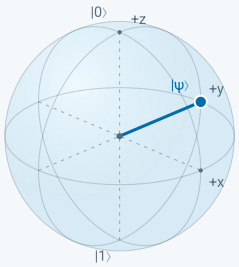
This explorer deliberately models pure states only. The projected ellipse and arrow are drawing choices; they do not depict a hardware qubit's literal shape.

Check yourself: identify the coordinates of $|-\rangle$. They are $(-1, 0, 0)$.

Read axes before interpreting the perspective

WORKED DIAGRAM · F11

Positive y is the state with a positive imaginary component



Polar angle θ: 90°

Relative phase φ: 90°

α = 1/√2; β = i/√2

x = 0, y = 1, z = 0

For θ = 90° and φ = 90°, the Bloch vector is exactly (0, 1, 0). Its arrow and the +y axis use the same orthographic projection, so their endpoints coincide. The axes are a three-dimensional convention, not hardware wires.

This diagram uses an orthographic view of three perpendicular mathematical axes. Positive z points upward, positive x projects downward and right, and positive y projects upward and right. The state at polar 90 degrees and relative phase 90 degrees lies on positive y, so the state tip and the positive-y endpoint coincide. The perspective is a drawing convention; the coordinate formulas determine the physics.

All axes, great circles and state arrows are projected using the same transformation. This matters because a plausible-looking sphere can still teach the wrong sign if an arrow is projected differently from an axis. When checking a diagram, use the six reference states first. North and south should match the two Z states, while the four equatorial directions should agree with the signed X and Y states.

Worked conversion from amplitudes

For the positive-y state, the first amplitude is one over the square root of two and the second is i over the square root of two. The conjugate product of the first amplitude with the second is i over two. Doubling its real and imaginary parts gives x equal to zero and y equal to one. Subtracting the squared magnitudes gives z equal to zero.

Now apply H. The output amplitudes become one half plus i over two and one half minus i over two. Their conjugate product has imaginary part negative one half, so the new Bloch vector points along negative y. The Z probabilities stay equal. This example checks the geometry and the amplitude convention together: H reverses the y direction rather than leaving every equatorial state unchanged.

For a second example, take positive real amplitudes square root of three over two and one half. The product is square root of three over four, giving x equal to square root of three over two. The z coordinate is one half. The corresponding polar angle is 60 degrees and relative phase is zero. The state lies above the equator on the positive-x side. You can recover the probabilities directly from z: zero has probability three quarters and one has probability one quarter.

Coordinates are expectation values

The x, y and z coordinates are the expected signed outcomes of the Pauli measurements. Assign the positive eigenvalue to the corresponding positive-axis state and the negative eigenvalue to its opposite. The z expectation is the

probability of zero minus the probability of one. This gives the relation between the vertical coordinate and the Z histogram without requiring a geometrical guess.

To estimate x experimentally, rotate into the X basis before readout. To estimate y, use an appropriate phase rotation and H before Z readout. Each setting needs many fresh preparations. You do not obtain all three coordinates as exact numbers by measuring one physical qubit. The simulator can show them simultaneously because it already knows the prepared ideal state.

Pure states are the surface, mixtures the interior

A pure qubit has Bloch-vector length one. A mixed state can have a shorter vector. An equal mixture of zero and one lies at the centre: all three Pauli expectations vanish. It is not the equatorial plus state, whose x expectation is one. Both give equal Z probabilities, so their different locations encode information a Z histogram misses.

The lab prepares only pure states and applies ideal unitary gates, so its state tip remains on the surface. It cannot illustrate decoherence merely by drawing a moving pure-state arrow. A model of dephasing would shorten appropriate ensemble components, requiring a mixed-state description. Keep this limitation in mind when interpreting the sphere as a picture of a noisy device.

Check the positive-y reference yourself

The explorer opens at polar 90 degrees and relative phase 90 degrees. Before revealing, predict the amplitudes, the equal Z probabilities and the coordinate vector. Reveal and check that the arrow reaches the marked positive-y endpoint. Then apply H and predict the negative-y vector. The amplitudes may have a different common phase from a familiar written representative, but the coordinates should agree.

L01 · INTERACTIVE LAB

Runs on your device

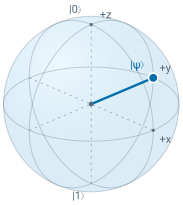
Qubit explorer

Prepare a pure qubit, apply gates, then compare amplitudes, phases and Z-basis probabilities. Reset restores the initial preparation shown by the lesson's controls.

MODEL · Ideal pure-state vector, complex double precision

ORDER · q0 is the leftmost bit

Initial setup · reference state



Exact probability

|0⟩ 50.0%

|1⟩ 50.0%

Statevector before readout · ideal simulation

Basis	Amplitude a + bi	Magnitude	Phase (rad)	Probability
|0⟩	`0.707107 + 0i`	0.707107	0.000000	50.0%
|1⟩	`0 + 0.707107i`	0.707107	1.570796	50.0%

Signs and complex phases are part of the state. Phase is undefined for a zero amplitude. A shared global phase changes no probabilities; relative phases can affect a later operation. Values are rounded to six decimal places.

Prediction to try: Predict the amplitudes and Z-basis probabilities for the preparation and gates you have chosen. If starting from |0⟩, what will H do?

Challenge: Prepare θ = 120°, φ = 90°. Predict both complex amplitudes. Then change φ without changing θ.

Open the live experiment studio

Reset, then move the relative phase through zero, 90, 180 and 270 degrees at the equator. State each expected signed axis before revealing. Finally move the polar angle to zero and vary the phase. The arrow should remain at the north pole because the phase of a zero second amplitude has no physical effect.

Practice: reconstruct probabilities from geometry

A state has Bloch coordinates zero, square root of three over two, and negative one half. Find its two Z probabilities. Is it pure? Give one compatible amplitude representation with a positive real zero component.

Check your reasoning

The probabilities are one quarter for zero and three quarters for one. The squared coordinate length is three quarters plus one quarter, so the state is pure. A compatible vector has amplitudes one half and i times square root of three over two. The positive y coordinate fixes the sign of the imaginary relative component in this convention.

Reference: Microsoft Learn — The qubit.

Check your understanding

PAUSE & THINK

At θ = 90° and φ = 90°, which Bloch direction is correct?

- ☐ +x
- ☐ +y
- ☐ −y

Answer: +y. The coordinates are (0,1,0), so the state tip must lie on the positive-y endpoint.