Entanglement and Bell states

A joint quantum state can contain relationships no product of individual states can describe.

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By the end, you'll be able to

- Construct a Bell state using H and CX.
- Distinguish entanglement from independent states and classical correlation.
- Explain why entanglement does not send a message faster than light.

Build a Bell pair

Start from $|00\rangle$. Apply H to q0, then controlled-X with q0 as control:

$$|00\rangle \xrightarrow{H_0} \frac{|00\rangle + |10\rangle}{\sqrt{2}} \xrightarrow{CX_{0,1}} |\Phi^+\rangle = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

The Z-basis outcomes are 00 and 11, each with probability $1/2$. Each individual outcome is random, but the pair agrees.

Why it is not a product

Suppose $(a|0\rangle + b|1\rangle) \otimes (c|0\rangle + d|1\rangle)$ equalled this Bell state. We would need ac and bd nonzero, but $ad = bc = 0$. Nonzero ac and bd require all four factors nonzero, contradicting those zeros. The state is not separable.

One basis is not enough evidence

A classical 50/50 mixture of 00 and 11 has the same Z-basis counts as this Bell state. Their X-basis behaviour differs: the ideal Bell state still has matching outcomes, while that classical mixture does not preserve those X correlations.

The lab compares Bell preparation with independent $|+\rangle$ states. This distinguishes those two known ideal preparations; it is not a general experimental certification of an arbitrary unknown device. Such a claim needs carefully chosen observables and assumptions.

Correlation is not communication

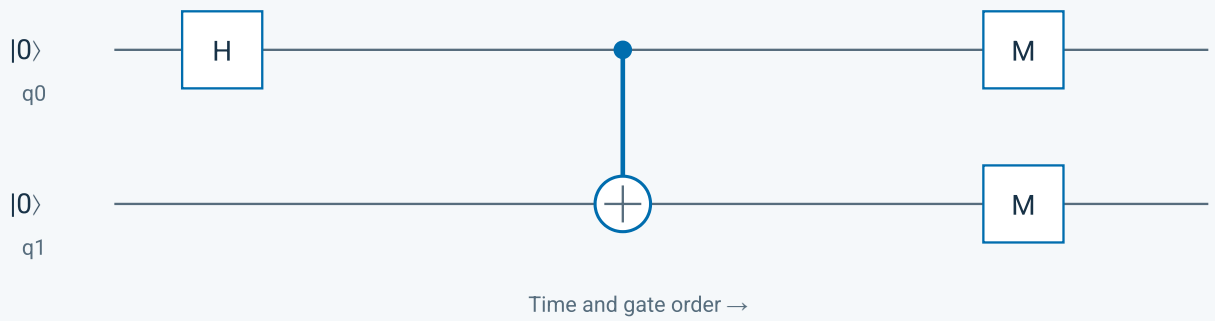
Measuring one half gives a random local outcome. The other reader cannot use their own outcomes to learn which measurement was chosen remotely. Comparing correlations requires ordinary classical communication.

Challenge: predict the independent-state distributions in Z and X before switching the selector. Explain both using the product-state structure.

Trace the preparation one wire at a time

WORKED DIAGRAM · F13

Prepare and inspect a Bell pair



H acts on the upper, leftmost qubit q0. CX has q0 as control and q1 as target. A Z readout gives only 00 or 11; other bases are needed to distinguish this coherent state from a classically correlated mixture.

The H box acts on q0, the upper and leftmost-labelled qubit. The filled dot is the CX control and the circled plus is its target. Starting from 00, H creates amplitudes on 00 and 10. CX leaves the first branch unchanged and turns 10 into 11. The output therefore has two nonzero joint amplitudes, both positive and equal.

Notice that the control is not measured. The gate applies its reversible rule to every component coherently. If you replaced the control by a classical measurement followed by a conditional X, you could reproduce the Z correlations but would generally lose the Bell state's coherence. The distinction appears when you change the readout basis or attempt to reverse the preparation.

Worked proof that the state is not a product

Suppose the Bell state were a product of two one-qubit states. Its amplitudes would have the form ac, ad, bc and bd. Matching the desired 00 and 11 amplitudes requires ac and bd to be nonzero. That means all four factors are nonzero. But then ad and bc are also nonzero, contradicting the required zero amplitudes on 01 and 10.

Equivalently, reshape the amplitudes into a diagonal two-by-two array with both diagonal entries one over the square root of two. Its determinant is one half. A product state has determinant zero. This is a compact pure-state separability test for two qubits, not a complete mixed-state entanglement test. State the assumptions before applying it to experimental data.

Compare three preparations

An independent plus-plus preparation gives all four Z outcomes equally often. A Bell preparation gives only agreeing Z outcomes. A classical mixture that prepares 00 or 11 with equal probability also gives only agreeing Z outcomes. The first comparison detects correlation, but the second shows that correlation alone does not prove entanglement.

Now measure both qubits in X by applying H to each before Z readout. The Bell state still gives agreeing results. The classical 00/11 mixture gives all four X outcome pairs equally. An independent plus-plus state is deterministic in X,

giving 00. These three predictions provide a useful small comparison table across bases. They are predictions for specified ideal model states; certifying an unknown device needs a fuller experimental and statistical argument.

The lab compares the Bell and independent preparations directly. It does not include a general mixed-state simulator. Use the classical-mixture calculation on paper to understand why the two displayed preparations are not the only possible explanations for a Z histogram. A tool's menu is not the whole space of physical hypotheses.

Local randomness does not allow signalling

For a Bell pair, either participant's Z result is individually random. A remote participant can choose a local operation or measurement, but without receiving the remote outcome the local observer's statistics do not reveal a controllable message. The striking correlations appear when the participants later compare records through an ordinary classical channel.

If one party measures and communicates the result, the other can predict the matching result in the same basis. That useful conditional knowledge does not imply faster-than-light communication, because receiving the condition required a classical message. Distinguish a joint probability statement from information available locally before the records are compared.

Entanglement is also not an automatic computational speedup. Some entangled circuits are efficiently simulable with specialised classical methods. An algorithmic advantage depends on the full circuit, access model, desired output and best classical comparison. Here the Bell pair is a compact example of a non-product state and a resource used by later communication protocols.

Test preparation and basis separately

Start with the default Bell preparation and Z readout. Record the expected nonzero bitstrings and reveal. Then change only the basis to X and predict again. The amplitude table includes the H readout rotations, so it describes the state immediately before the final Z detector rather than the state before those rotations.

L04 · INTERACTIVE LAB

Runs on your device

Entanglement explorer

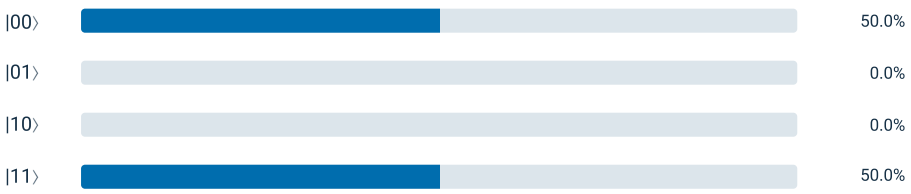
Compare a Bell preparation with independent |+⟩ states. An X readout applies H to each qubit before Z measurement.

MODEL · Ideal pure-state vector, complex double precision

ORDER · q0 is the leftmost bit

Initial setup · reference state

Exact probability



Statevector before readout · ideal simulation

Basis	Amplitude a + bi	Magnitude	Phase (rad)	Probability
|00⟩	`0.707107 + 0i`	0.707107	0.000000	50.0%
|01⟩	`0 + 0i`	0.000000	undefined	0.0%
|10⟩	`0 + 0i`	0.000000	undefined	0.0%
|11⟩	`0.707107 + 0i`	0.707107	0.000000	50.0%

Signs and complex phases are part of the state. Phase is undefined for a zero amplitude. A shared global phase changes no probabilities; relative phases can affect a later operation. Values are rounded to six decimal places.

Prediction to try: For this preparation and readout basis, which bitstrings should occur? Will the two recorded bits agree?

Challenge: Compare both preparations in both bases. Evidence about these known model states is not a general certification of an unknown device.

Open the live experiment studio

Switch to the independent preparation and repeat both bases. Explain the change in terms of the prepared joint state. Use fresh predictions for each setting and record the preparation, basis and shot count alongside any sampled frequencies. A statement such as “the experiment produced correlated bits” is incomplete without those settings.

Practice: reverse the Bell circuit

Starting from the Bell state, apply CX with the same control and target, then H on q0. Calculate the output. Compare that with applying H first and CX second.

<details>
<summary>Check your reasoning</summary>

CX maps the 11 branch back to 10, giving plus tensor zero. H on q0 then returns 00. The inverse must reverse the preparation order. Applying H first is a different circuit and does not implement that inverse. Reversal is an informative coherence check because a classical 00/11 mixture does not return a definite 00 under the same inverse circuit.
</details>

Reference: Microsoft Learn — Entanglement and correlations.

Check your understanding

PAUSE & THINK

Do agreeing Z outcomes alone prove a Bell state was prepared?

- [ ] Yes, always
- [ ] No; a classical 00/11 mixture gives the same Z distribution
- [ ] Only with exactly equal counts

Answer: No; a classical 00/11 mixture gives the same Z distribution. Additional phase-sensitive settings are needed. Matching Z correlations do not uniquely identify the joint state.