# Quantum reversibility and the no-cloning principle

Ideal gates are reversible. Copying an arbitrary unknown state is a different request.

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**By the end, you'll be able to**

- Reverse a sequence of unitary gates.
- Give the inner-product argument against universal perfect cloning.

## Undoing a gate

An ideal unitary $U$ has inverse $U^\dagger$. To reverse a sequence, reverse its order and invert each gate. If a circuit applies H, then S, its state is $SH|\psi\rangle$. Undo it by applying $S^\dagger$ first and H second.

X, Z, H and CX are their own inverses. S is not: $S^\dagger = \mathrm{diag}(1, -i)$.

## Copying known basis information

CX maps $|0\rangle|0\rangle$ to $|00\rangle$ and $|1\rangle|0\rangle$ to $|11\rangle$. This copies a known computational-basis label into a fresh zero target.

For a superposition, however,

$$CX\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} \otimes |0\rangle\right) = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

That is an entangled Bell state, not $|+\rangle \otimes |+\rangle$.

## Why no universal perfect cloner exists

Suppose a unitary copied both unknown states $|a\rangle$ and $|b\rangle$ into a blank target. Preservation of inner products would require

$$\langle a|b\rangle = (\langle a|b\rangle)^2.$$

This only holds when the overlap is 0 or 1. It fails for general distinct, nonorthogonal states. A single universal operation therefore cannot perfectly clone arbitrary unknown pure states. This does not forbid preparing many copies of a state whose description you already know.

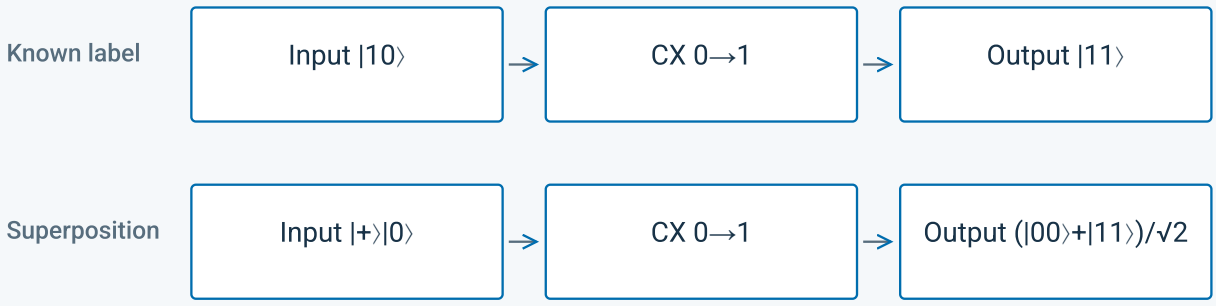
## Practise

Prepare a Bell state in the gate playground. Undo CX and then H. Verify that all probability returns to 00. Explain why measurement halfway through would change this reversal experiment.

## Reversibility preserves distinctions

WORKED DIAGRAM · F14

### Copying basis labels is not universal state cloning




CX copies a computational-basis label into a zero target. With a plus-state control it creates a Bell state, whereas two independent plus states have four nonzero basis amplitudes.

A reversible operation cannot map two different full inputs to the same full output. Classical irreversible instructions such as erasing a bit hide information in an environment or discard it from the computational description. A unitary gate preserves inner products and therefore preserves distinguishability. This is why a reversible function implementation usually keeps enough input or workspace to undo the calculation.

Reversible does not mean that every desired transformation is available. A universal cloning machine would need to preserve the original unknown state while creating another identical one. That extra requirement conflicts with linearity and inner-product preservation. Known states can be prepared repeatedly from their descriptions, and orthogonal basis labels can be copied; neither operation is universal cloning of an unknown input.

## Worked example: CX copies a label, not a qubit

Start with an arbitrary first-qubit state and a zero target. CX gives a two-term joint state:

$$(\alpha|0\rangle + \beta|1\rangle)|0\rangle \longmapsto \alpha|00\rangle + \beta|11\rangle.$$

Two independent copies of the original state would instead have four amplitudes, including cross terms proportional to the product of alpha and beta. For a plus input, the desired two-copy state is plus tensor plus, whose four amplitudes are all one half. The actual CX output is a Bell state with only two nonzero amplitudes. Their Z statistics are already different: the Bell outputs always agree, while independent plus copies sometimes disagree.

For basis inputs, one of the two amplitudes vanishes and the difference disappears. CX correctly copies the zero or one label into the target. A device can therefore copy a chosen orthogonal alphabet while failing on superpositions of that alphabet. Testing only basis examples would not establish that a proposed operation copies arbitrary quantum information.

## A short no-cloning argument

Assume a unitary copies two possible normalised states into a blank target. Let their overlap be s. Before copying, the full inputs have overlap s because the blank targets are identical. After perfect copying, the outputs have overlap s squared. Unitarity requires those overlaps to agree, so s must equal s squared. The only possibilities are zero or one: orthogonal states or the same physical state after consistent phase choices.

Take zero and plus as concrete candidates. Their overlap is one over the square root of two, which is neither zero nor one. A single deterministic unitary cannot clone both perfectly. This proof does not say that all approximate copying is forbidden, nor that a known classical preparation recipe cannot be reused. It identifies a precise impossible operation: universal perfect copying of an unknown quantum state.

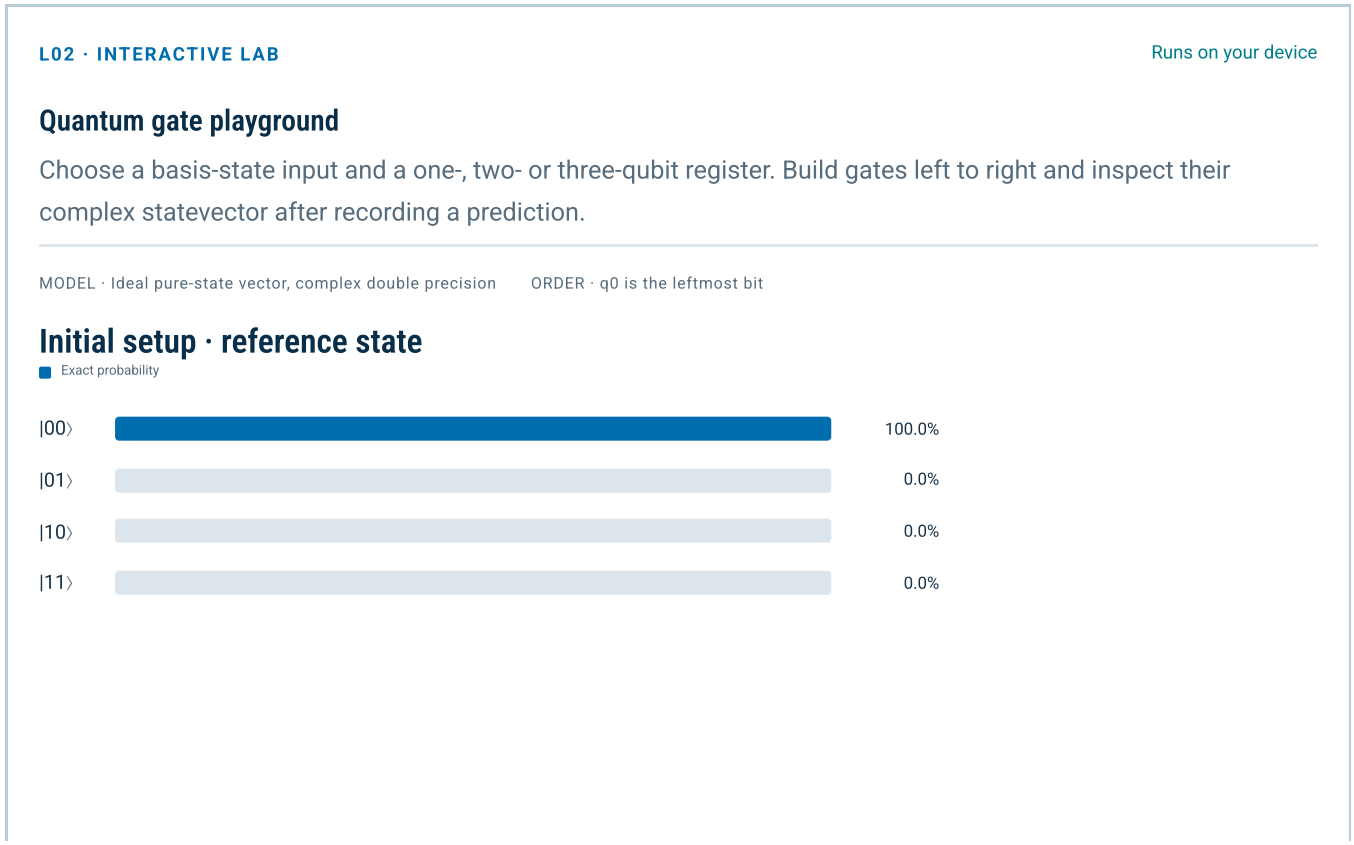
## Undo a circuit in reverse order

Suppose a circuit applies H, then S, then CX. To undo it, apply CX first, then the inverse of S, then H. H and CX are self-inverse. The inverse of S can be implemented by three S gates, since four S gates make the identity. Applying the same original sequence again generally does not undo it, because order and inverses both matter.

This is also the basis of uncomputation. A reversible subroutine may leave workspace correlated with its input. After using a computed value appropriately, an inverse subroutine can return that workspace to a standard state. Simply ignoring unwanted workspace may destroy coherence in the subsystem you keep. Reversible algorithm design therefore includes a plan for temporary information, not just a forward list of arithmetic gates.

## Compare copying and independent preparation

Use the default two-qubit zero input. Add H to q0 and CX from q0 to q1. Predict the Bell state and reveal it. Then reset, apply H to each qubit and predict the independent plus-plus state. Compare all four complex entries, not just whether each individual bit looks random.



**Statevector before readout · ideal simulation**

| Basis | Amplitude a + bi | Magnitude | Phase (rad) | Probability |
|---|---|---|---|---|
| \|00⟩ | `1 + 0i` | 1.000000 | 0.000000 | 100.0% |
| \|01⟩ | `0 + 0i` | 0.000000 | undefined | 0.0% |
| \|10⟩ | `0 + 0i` | 0.000000 | undefined | 0.0% |
| \|11⟩ | `0 + 0i` | 0.000000 | undefined | 0.0% |

Signs and complex phases are part of the state. Phase is undefined for a zero amplitude. A shared global phase changes no probabilities; relative phases can affect a later operation. Values are rounded to six decimal places.

Prediction to try: For the chosen input and circuit, predict the nonzero amplitudes, their signs and the measured bitstrings.

Challenge: Compare H|0⟩ with H|1⟩ using one qubit. Their Z probabilities agree; their amplitude signs do not. Add a second H to distinguish them.

Open the live experiment studio

For a reversal trial, build H on q0 followed by S on q0. Add three more S gates and then H. Predict the final zero state before revealing. Explain why the extra S gates implement an inverse rather than a new measurement. If you extend the circuit to both wires, identify which subsystem every gate acts on.

## Practice: copying a known recipe

An engineer knows the exact angles used to prepare a qubit and uses that classical recipe to prepare one hundred identical systems. Does this violate no-cloning? How does it differ from receiving one unknown qubit and being asked to duplicate it without any preparation information?

**Check your reasoning**

Reusing a classical preparation description does not violate no-cloning. The engineer has a recipe and fresh hardware, not a universal operation extracting an unknown state from one specimen. The unknown-input task lacks that classical information. Measurements on one copy cannot reveal arbitrary continuous amplitudes exactly, and a deterministic universal unitary copier is ruled out by the overlap argument.

Reference: IBM Quantum — Basics of quantum information.

## Check your understanding

PAUSE & THINK

### Does CX on |+⟩|0⟩ make two independent plus states?

- [ ] Yes
- [ ] No; it makes a Bell state
- [ ] Only if it is followed by a Z measurement

**Answer:** No; it makes a Bell state. Its output is (|00⟩+|11⟩)/√2, which differs from the four-term product |+⟩|+⟩.