Quantum teleportation

Transfer an unknown qubit state using a shared Bell pair and two ordinary bits.

Core 18 min lesson Dr. Charith Jayasekara

By the end, you'll be able to

- Trace all four measurement branches and their corrections.
- Explain the shared entanglement and classical communication resources.
- Test preservation of relative phase.

Before you begin: Qubit states, H and X/Z gates, CNOT, Bell pairs, measurement.

Move the state, using shared resources

Suppose a quantum sensor prepares a qubit that another processor needs. Measuring it once gives one outcome, so sending that outcome cannot preserve all its quantum information. Teleportation solves a different task: it transfers the state to an existing receiver qubit, using entanglement shared earlier and two classical bits sent afterwards.

Think of the sender and receiver as two project teams. They prepare a shared resource before the task begins, then a short message tells the receiving team which correction completes the task. This analogy describes coordination; the shared resource here is a quantum Bell pair, whose correlations must be described by the circuit.

The input can be any normalised qubit:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \qquad |\alpha|^2 + |\beta|^2 = 1.$$

Alice holds the input Q and an auxiliary qubit A. Bob holds B. Initially, A and B share

$$|\Phi^+\rangle_{AB} = \frac{|00\rangle + |11\rangle}{\sqrt{2}}.$$

Checkpoint. Which two systems must already be entangled? A and B. The unknown input Q is a separate system at the start.

Follow the three-wire circuit

Use the fixed wire order Q, A, B, from top to bottom. Prepare the Bell pair with H on A, followed by CNOT from A to B. Alice then applies CNOT from Q to A, followed by H on Q. She measures Q to obtain m, and A to obtain n. She sends both bits to Bob. Bob applies X when $n = 1$, then Z when $m = 1$.

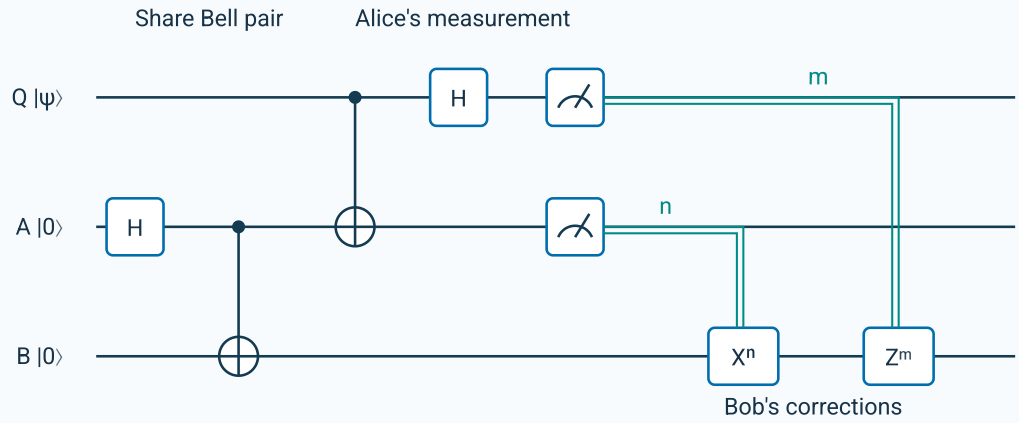
The important identity is that Alice's two measurements select one of four correctable branches:

$$\frac{1}{2} \sum_{m,n\in\{0,1\}} |mn\rangle_{QA}\, X^n Z^m |\psi\rangle_B.$$

Each pair mn has probability $1/4$, regardless of α and β. Applying X^n and then Z^m means the combined correction is $Z^m X^n$, which reverses the branch operation.

Alice's bits mn	Bob's state before correction	Apply, in time order
00	$	\psi\rangle$
01	$X	\psi\rangle$
10	$Z	\psi\rangle$
11	$XZ	\psi\rangle$

Teleportation: one input, four correctable branches



Classical bits	00	01	10	11
Before correction	|ψ⟩	X|ψ⟩	Z|ψ⟩	XZ|ψ⟩
Probability	¼	¼	¼	¼

Each correction restores the same state |ψ⟩ on B.

Three-wire teleportation circuit with classical corrections and all four branch probabilities.

The doubled lines carry classical bits. The lower quantum output carries the restored input state; the upper two measurements choose its correction.

Worked example: Work one branch by hand

Choose an input with unequal probabilities and a visible complex phase:

$$|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{i}{2}|1\rangle.$$

If Alice obtains $mn = 11$, Bob's state is

$$XZ|\psi\rangle = -\frac{i}{2}|0\rangle + \frac{\sqrt{3}}{2}|1\rangle.$$

After X, it becomes $\sqrt{3}|0\rangle/2 - i|1\rangle/2$. After Z, it becomes the original input. Thus both the magnitudes and the relative phase survive. A Z-basis test alone would miss many phase errors, so a simulator should compare the complete state or measure several bases on repeated preparations.

Explore and check

In an ideal simulator, freely choose an input, then inspect all four branches. Keep your input visible while switching branches. Temporarily omit each correction and compare the output in X, Y and Z bases. For the input $|+\rangle$, omitting the phase correction can turn $|+\rangle$ into $|-\rangle$: their Z histograms match, while their X outcomes differ completely.

Before Alice's message arrives, Bob's state averaged over the unknown outcomes is $I/2$. Therefore he cannot read the input or a message from his qubit alone. Alice's measurements also consume the original quantum state and the Bell pair. Resource accounting is one shared Bell pair plus two transmitted classical bits per ideal teleported qubit. This explains both the communication requirement and why the process produces one usable output state.

Exercises

1. For input $|0\rangle$ and bits 01, find Bob's state before and after correction.
2. Why is testing only $|0\rangle$ and $|1\rangle$ insufficient?

<details>
<summary>Worked answers</summary>

1. $|1\rangle$, then $|0\rangle$ after X.
2. phase mistakes can preserve both; also test superpositions such as $|+\rangle$ and $(|0\rangle + i|1\rangle)/\sqrt{2}$.

</details>

Check your understanding

PAUSE & THINK

Alice measures m=1,n=0. What should Bob do?

- [ ] Apply Z
- [ ] Apply X
- [ ] Measure immediately and send the result back

Answer: Apply Z. The uncorrected branch is Z|ψ⟩ and Z²=I.

Teleportation transfers quantum information through a shared entangled resource and a classical message. The four branch equations explain why every measurement outcome can succeed.

Sources

Bennett et al., original teleportation paper, 1993; IBM Quantum Learning, teleportation. The worked state, activities and figure above are original teaching examples.